**2 a)**

The probability of all 5 variables being true is  
P(D=True|A=True,B=True)\*P(E=True|B=True,C=True)\*P(A=True)\*P(B=True)\*P(C=True)  
= 0.2\*0.5\*0.8\*0.1\*0.3 = 0.0024

**2 b)**

The probability of all 5 variables being false is  
P(D=False|A=False,B=False)\*P(E=False|B=False,C=False)\*P(A=False)\*P(B=False)\*P(C=False)  
= 0.1\*0.8\*0.8\*0.5\*0.2 = 0.0064

**2 c)**

We have to find   
P(A=F | B,D,E,C) = α \* P(A,B,D,E,C)  
= α \* P(A=F) \* P(B=T) \* P(C=T) \* P(D=T | A=F, B=T) \* P(E=T|B=T,C=T)  
= α \* 0.8 \* 0.5 \* 0.8 \* 0.6 \* 0.3 = 0.0576 \* α  
Also,  
P(A=T | B,D,E,C) = α \* P(A,B,D,E,C)  
= α \* P(A=T) \* P(B=T) \* P(C=T) \* P(D=T | A=T, B=T) \* P(E=T|B=T,C=T)  
= α \* 0.2 \* 0.5 \* 0.8 \* 0.1 \* 0.3 = 0.0024 \* α  
α = 1/(0.06) = 0.96  
So,   
P(A=F | B,D,E,C) = 0.0576 \* 0.96 = 0.055296

**3. a)**

From the Bayesian network we get,  
P(B | J=True, M=True)   
= α \*P(B)\* e∑ P(E) \* a∑ P(A|B, E) \* P(J=True| A) \* P(M=True| A)  
= α \*P(B)\* e∑ P(E) \* { P(A=True| B, E) \* P(J=True| A=True) \* P(M=True| A=True)  
 + P(A=False| B, E) \* P(J=True| A=False) \* P(M=True| A=False) }  
= α \*P(B)\* [ P(E = True) \* { P(A=True| B, E) \* P(J=True| A=True) \* P(M=True| A=True)  
 + P(A=False| B, E) \* P(J=True| A=False) \* P(M=True| A=False) }  
 + P(E = False) \* { P(A=True| B, E) \* P(J=True| A=True) \* P(M=True| A=True)  
 + P(A=False| B, E) \* P(J=True| A=False) \* P(M=True| A=False) }]  
Also,  
P(~B| J=True, M=True)   
= α \*P(~B)\* e∑ P(E) \* a∑ P(A|B, E) \* P(J=True| A) \* P(M=True| A)  
= α \*P(~B)\* e∑ P(E) \* { P(A=True| ~B, E) \* P(J=True| A=True) \* P(M=True| A=True)  
 + P(A=False|~ B, E) \* P(J=True| A=False) \* P(M=True| A=False) }  
= α \*P(~B)\* [ P(E = True) \* { P(A=True|~ B, E) \* P(J=True| A=True) \* P(M=True| A=True)  
 + P(A=False| ~B, E) \* P(J=True| A=False) \* P(M=True| A=False) }  
 + P(E = False) \* { P(A=True|~ B, E) \* P(J=True| A=True) \* P(M=True| A=True)  
 + P(A=False| ~B, E) \* P(J=True| A=False) \* P(M=True| A=False) }]  
Solving for the two equations using the Probabilities given,  
P(B|J=True, M=True) = 0.2841

**3 b)**   
As we can see above with enumeration the total number of operations is 14+14 = 28.   
With variable elimination we can do this in 8 + 8 = 16 operations.

**3 c)**For enumeration the each node in the enumeration tree has 2 branches hence the time complexity is O(2^n).  
With variable elimination since at each level we are storing the intermediate results, and each level the branching factor is two. The total number of operation will reduce by a factor of 2. So the order is O(2^n-1).

**5 a)**The expected net gain = P(q+) \* U(q+) + P(q-)\*U(q-)  
= 1000 \* 0.7 + (-400) \*0.3   
=580

**5 b)**P(pass | q+) = P(q+ | pass) \* P(pass) / P(q+)  
With the given values P(q+|pass) \* P(pass) = 0.8 \* 0.7 = 0.56  
P(pass | q-) = P(q- | pass) \* P(pass) / P(q-)  
With the given values P(q-|pass) \* P(pass) = 0.35 \* 0.3 = 0.105  
Therefore,  
P(q+|pass)/P(q-|pass) = 0.56/0.105 = 5.33 ( Since, sum of probabilities is equal to 1)  
Which implies,  
**P(q+|pass) = 0.84**,  
We can now derive, **P(pass) =0.67, P(q-|pass) = 0.16**  
and also **P(fail) = 0.33**  
  
Now,  
P(fail | q+) = P(q+ | fail) \* P(fail) / P(q+)  
0.2 = P(q+ |Fail) \*0.33 / 0.7  
**P(q+|Fail) = 0.42**  
Also, **P(q-|Fail) = 0.68**

**5 c)**The expected utility of pass is  
= P(q+ | pass) \*U(q+) + P(q-|pass)\*U(q-)  
= 0.84 \* 900 + (-500) \* 0.16  
= 685  
The expected utility of fail is  
= P(q+ | fail) \*U(q+) + P(q-|fail)\*U(q-)  
= 0.42 \* 900 + 0.58 \* (-500)  
= 88  
Since, in both the cases the utility is positive, the best decision is to buy in both the cases.

**5 d)**The net gain with the information is  
= P(pass) \* U(pass) + P(fail) \* U(fail)  
=0.67 \* 685 + 0.33 \* 88  
= 488  
This is less than the expected net gain if we buy without taking to the mechanic(=580). Hence, we should not take to the mechanic.